Error Quantification in Oriented-Core Data and its Influence on Rock Slope Design

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Abstract

Rock slope design requires detailed knowledge of the rock joint fabric, which is the most important parameter in bench-scale analysis. Rock-fabric data for open pit slope design may be difficult to obtain because mine planning often requires design slope angles years in advance of excavation and outcrop exposure. When rock-fabric data cannot be obtained from surface mapping, one of the most cost-effective alternatives to obtain these data is oriented-core drilling. Because of various uncertainties in the core orienting process and the scale effect of measuring orientations over the length of a core diameter, oriented-core joint sets typically display greater dispersion than do the same sets from surface mapping. This paper presents a method to quantify some of the errors induced during the core orientation process with a simple measurement taken by the technician at the time of core logging, called the difference angle. The error distribution is quantified from the distribution of the difference angles and removed from the measured data population. The result is an improved representation of the in situ variation on the joint set. This reduction in data scatter is particularly important when conducting probabilistic bench-scale analysis with a joint set whose dip is near the joint friction angle. An example is given where the design slope interramp angle is increased by two degrees as a result of the reduction in the variance of the oriented-core data utilizing this method.

1 Introduction

Joint orientation data is one of the most important input parameters in hard-rock slope design. It is the most influential criterion for rock bench design, and can mandate the need for anisotropic rock-mass strengths in overall rock slope analysis. Rock-fabric data is preferably obtained from surface mapping of outcrop exposure (Park & West 2002, Sullivan et al. 1992). However, design slope angles are at times required years in advance of excavation when there is little or no rock exposure available in the location of interest. Even if surface mapping data are available, there is no guarantee that rock-fabric data collected on the surface will correspond to that at depth. Often times the most cost effective method to obtain the needed joint orientation data for rock slope design is through oriented core drilling (weighted core barrel, scribe, accelerometer, etc.).

Joint sets obtained from oriented-core drilling typically are more dispersed than the same sets obtained through surface mapping. The reason for this is twofold. First, a scale effect is introduced when measuring the orientation of a joint over the length of a core diameter. Joint waviness adds dispersion to the data compared to surface mapping where average orientations can be measured over lengths typically greater than one meter. Second, core orientation is a multi-step process where each step has the potential to introduce error. The first of these sources is impossible to address without information on the waviness of the joint sets being sampled; very little research has been completed to address the second. Methods have been proposed to classify potentially unreliable oriented-core data at the time of drilling (2iC Australia Pty. 2006) and during the data processing phase (Holcombe et al. 2011, Nelson 1987), but little has been introduced regarding quantification of error induced by the core orientation process itself.

A measurement called the difference angle can be used to quantify some of the error induced during the core orientation process. If it is assumed the mean vector of a joint set is correctly identified from oriented-core
drilling, the calculated distribution of error can be statistically removed from the joint set data population. The result is a joint set with less dispersion, which can alter the achievable interramp angle predicted by probabilistic rock bench analysis.

2 Oriented-core data collection and interpretation

2.1 The difference angle

The difference angle (DA) is the absolute value of the difference in measured in situ core orientation from two consecutive drilling runs. It is measured at the time of core orientation with a goniometer or protractor, and can be used to quantify the distribution of some of the error induced during those drilling runs. Under ideal conditions with no error, the DA will always be zero. It is a measure not of absolute error, but of relative error from one core run to the next.

A flow chart of the core orientation process is presented in Figure 1. The DA measures the distribution of total error induced by the steps highlighted in red (Fig. 1).

![CORE ORIENTATION FLOW CHART]

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The distribution of error induced by the processes shown in RED is measured by the difference angle.

Data scatter is also induced by:
• Natural joint set variation
• α-angle measurement error
• β-angle measurement error
• Down-hole survey error
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Figure 1. Flow chart of a typical core orientation process.

2.2 The reference, alpha, and beta angles

There are many conventions to measure the three angles required for joint orientations. This paper adopts the convention suggested by Call (1980): The reference, alpha (α), and beta (β) angles. These three angles, along
with the azimuth and plunge of the drill hole at the location the joint, are required in order to calculate a joint’s *in situ* orientation. These three angles are presented graphically in Figure 2, and defined as:

- **Reference angle** – The circumferential angle measured around the core between the reference line and the mark representing the *in situ* orientation of the core (top or bottom of hole) measured counter-clockwise, looking in the direction of drilling.
- **α-angle** – The angle between the joint surface and the core axis.
- **β-angle** – The circumferential angle measured around the core from the tip of the discontinuity furthest down-hole counter-clockwise to the reference line, looking in the direction of drilling.

![Figure 2. Stick of oriented core demonstrating the relationship between the reference (yellow), alpha, and beta angles.](image)

### 2.3 Sources of error in the core-orientation process

The DA accounts for the distribution of cumulative error induced by measuring of the core orientation, marking of the orientation on the core, assembling the core run (twist), marking of the reference line, and measuring of the DA (Fig. 1). All sources of error measured by the DA can be considered positive or negative, are cumulative, and induce error only in the β-angle measurement.

Other sources of error include human error associated with the physical measurement of the α-angle and β-angle and down-hole survey error. Data scatter is also increased by *in situ* joint variation and joint waviness. The reproducibility of α-angle and β-angle measurements using a goniometer was investigated, and results obtained were similar to those by Nelson (1987): ±2.5° for α-angle measurements and ±3.5° for β-angle measurements.

To obtain accurate orientations, with any method, it is critical to maintain good relations and open communication with the driller, use precision instrumentation, and carefully measure angles to within a degree.

### 2.4 α-angle contours (isogonic lines)

The only error in the α-angle is in the physical measurement itself. Therefore, even with random β-angles (bad orientations) the possible orientation of a joint surface is constrained by the α-angle. For a given α-angle, the complete range of poles for different β-angles appears on a Schmidt net as a circle centered at the line of the drill hole (Terzaghi 1965, Priest 1985). An α-angle of 90° represents a joint perpendicular to the drill hole, in which case the β-angle is irrelevant. A full range of α-angle contours is presented in Figure 3a for a drill hole azimuth of 142° and plunge of -68° to demonstrate this concept. Key observations from Figure 3a include:

- Error in the β-angle can cause error in the calculated dip, dip direction, or both depending where on the α-angle contour it is located.
The magnitude of error in dip and dip direction induced by a given error in the $\beta$-angle is dependent on the $\alpha$-angle. The $\beta$-angle is less significant for large $\alpha$-angles, and very significant for small $\alpha$-angles.

Figure 3. (a) Lower hemisphere Schmidt net projection with drill hole line and corresponding oriented-core $\alpha$-angle contours, (b) A damaged joint with $\alpha = 10^\circ$, (c) An intact joint with $\alpha = 10^\circ$.

### 2.5 The blind zone and biased spacing zone

When joints are sampled along a linear scanline, such as in a drill hole, the frequency that joints are intersected relative to their true spacing will be biased (Terzaghi 1965). Joints that are non-orthogonal to the drill hole will be under-sampled such that joints parallel to the core axis ($\alpha = 0^\circ$) will theoretically never be sampled. This lack of sampling of joints near parallel to the core axis causes a void in the data where joints with an $\alpha$-angle less than $10^\circ$ would plot. The void in the data is referred to as the “blind zone”. The blind zone for a drill hole azimuth of $142^\circ$ and plunge of $-68^\circ$ is shaded red in Figure 3a.

Another source of data bias is typically introduced by the drilling and orienting process itself. Joints with $\alpha$-angles less than $20^\circ$ are commonly damaged by the drilling process (Fig. 3b) due to the long and thin pieces of core that are created by the small intersection angle of the joints with the core axis. It is very rare to encounter intact joints with an $\alpha$-angle less than $10^\circ$ (Fig. 3c). These damaged joints are typically bypassed by technicians (unless specifically instructed not to do so), introducing an additional spacing bias to joint sets in this region of the Schmidt net. This zone of spacing bias is shaded blue in Figure 3a.

### 2.6 Inspection for error in oriented-core data

New oriented-core data sets should always be inspected for excess error on a lower hemisphere Schmidt net during a drilling program when quality control can still be performed. A convenient method of inspection is to
plot the data assuming a vertical drill hole. This assumption centers the $\alpha$-angle contours relative to the net. Poles that form “circles”, or parts of circles centered relative to the net, indicate joint sets that have been randomized due to bad orientations. It is often useful to screen the data by rock type, technician, or RQD.

2.7 Oriented-core hole placement

Oriented core holes should be drilled perpendicular to the main joint set of interest. This is (1) to intersect the maximum number of joints possible for accurate spacing calculations, and (2) because the more perpendicular a joint set is to the drill hole, the less influence error in the $\beta$-angle will have on the calculated true orientations of joints. For geotechnical slope investigations, oriented core holes are typically drilled into final slope walls to best investigate for joints in the plane shear orientation.

3 Error quantification

3.1 Project background and bedding joint set

Three thousand meters of traditional oriented-core drilling utilizing electronic accelerometers for orientation was completed during a feasibility level geotechnical study for a large open pit mine. Of this drilling, 1848 meters was through a dark grey Cretaceous-age siltstone. The siltstone has predominant bedding and cross-bedding, which is variable due to gentle undulating folds. RQD is good to excellent.

One hundred ninety bedding joint measurements were obtained from oriented core hole OC1. The bedding joint set is presented on a truncated lower hemisphere Schmidt net in Figure 4a, and with the drill hole line and $\alpha$-angle contours in Figure 4b. The mean dip direction of the bedding set is $007^\circ$ and the mean dip is $16^\circ$. There is no apparent variation with depth. Bedding joint set properties are presented in Table 1. A negative exponential distribution is used to model both spacing and length. Spacing is estimated using the method described in Section 6.1 and lengths are estimated from cell mapping (Nicholas & Sims 2001, Call 1992) in the siltstone. Direct shear tests were performed on bedding joints from OC1 to determine a friction angle and cohesion.

Figure 4. (a) Siltstone bedding joint set poles plotted over truncated equal area Schmidt net, (b) Siltstone bedding joint set poles plotted over the drill hole line and truncated $\alpha$-angle contours.
Table 1. Bedding joint set properties

<table>
<thead>
<tr>
<th>Mean Spacing</th>
<th>Mean Length</th>
<th>Friction Angle</th>
<th>Cohesion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 m</td>
<td>9.7 m</td>
<td>22°</td>
<td>3 kPa</td>
</tr>
</tbody>
</table>

Mine planners propose to mine a nose near core hole OC1 which will expose a length of slope with a dip direction of 10°. An interramp design is required for 15 meter single benching. Since achieved rock catch-bench widths are variable, a probabilistic rock bench analysis will be performed. The bench design criteria will be an 80% reliability of achieving a 7.6 meter catch-bench width (Call 1992), and a 95% reliability of achieving a catch-bench width greater than 0 meters (a predicted catch bench of less than 0 meters indicates a bench failure large enough to affect the bench above).

3.2 Difference angle measurements

One hundred twenty-three DAs were measured during the drilling program from core runs within the siltstone (mean DA = 8.8°), which equates to approximately one DA measurement every five core runs. The probability density function (PDF) and cumulative distribution function (CDF) of the DA data are presented in Figure 5.

![Figure 5. PDF and CDF of measured difference angle distribution and normal distribution fit.](image)

3.3 Conversion from difference angle (relative error) to β-angle error (absolute error)

The DA is a measurement of relative error, and as such the quantification of the error measured by it must be treated statistically rather than deterministically. It is assumed that the summation of error induced by the core orientation process takes the form of a normal distribution (Koch & Link 1970) $P(X)$ defined by a mean ($\mu$) equal to zero, and an unknown variance ($\sigma^2$). A DA is generated by sampling this normal distribution $P(X)$ twice and taking the absolute value of the difference of the resultants. It can be shown that the distribution of the difference of two normally distributed independent variables $X_1$ and $X_2$ with means and variances ($\mu_{x1}, \sigma_{x1}^2$) and ($\mu_{x2}, \sigma_{x2}^2$), respectively is another normal distribution (Ang & Tang 1975) which has a mean and variance:
\[ \mu_{X_1+X_2} = \mu_{X_1} - \mu_{X_2} \]  
\[ \sigma^2_{X_1+X_2} = \sigma^2_{X_1} + \sigma^2_{X_2} \]

Since the same error distribution is sampled twice to generate a DA:
\[ \sigma^2_{X_1} = \sigma^2_{X_2} \]  
\[ \sigma^2_{X_1+X_2} = 2\sigma^2_{X} \]

Therefore if a normal distribution with \( \mu=0 \) can be fit to a population of measured DAs, the distribution of actual \( \beta \)-angle error \( P(X) \) is then also a normal distribution with \( \mu=0 \) and half the variance. Relationships are developed based on this between the mean measured difference angle, the mean induced error in the \( \beta \)-angle, and the resulting mean error in the final converted data, which is dependent on the \( \alpha \)-angle of a joint (Figs. 6a-6b).

Figure 6. Relationships between the mean difference angle and (a) the mean and median error induced in \( \beta \)-angle measurements, and (b) the mean error in the final converted data measured in dihedral angle

An exceptional fit to the DA data presented in Section 3.2 was obtained with \( \mu=0 \) and \( \sigma^2=124 \) (Fig. 5), which corresponds to a \( \beta \)-angle error distribution \( P(X) \) of \( \mu=0 \) and \( \sigma^2=62 \). A folded normal distribution (Nelson 1980) is used since the DA is always recorded as positive. The average quantified \( \beta \)-angle error for drilling in the siltstone is then \( 6.3^\circ \) and the median error is \( 5.3^\circ \) for core runs that included DA measurements.

### 3.4 Removal of measured \( \beta \)-angle error from a joint set population

#### 3.4.1 Required assumptions

It must be assumed that a mean vector for the joint set exists and that it is properly identified through oriented drilling. This is a reasonable assumption if the \( \beta \)-angle measurement error is randomly distributed and ample observations of the joint set are obtained.
3.4.2 Distribution of β-angle deviation from all sources

For each measurement, the difference is calculated between the β-angle that would have resulted in obtaining the mean vector of the joint set and the actual β-angle measured. This represents the deviation in the β-angle from the mean value. Before β-angles can be compared, they first must be converted to values that are equivalent (i.e. looking down-hole and reference angle of 0°). The distribution of deviation from the mean of the 190 siltstone bedding joint β-angle measurements is presented in Figure 7. A normal distribution with $\mu=0$ and $\sigma^2=471$ provides an excellent fit to the data (Fig. 7), and will be referred to as $P(X + Y)$.

Figure 7. CDF of deviation in β-angle measurements from the mean and normal distribution $P(X + Y)$, $\mu=0$ and $\sigma^2=471$.

3.4.3 Statistical reduction of joint set scatter

The distribution of the sum of two normally distributed independent variables $X$ and $Y$ with means and variances $(\mu_x, \sigma^2_x)$ and $(\mu_y, \sigma^2_y)$, respectively is another normal distribution (Ang & Tang 1975) which has a mean and variance:

\[ \mu_{X+Y} = \mu_X + \mu_Y \] \[ \sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y \]

It follows that,

\[ \mu_Y = \mu_{X+Y} - \mu_X \] \[ \sigma^2_Y = \sigma^2_{X+Y} - \sigma^2_X \]

Therefore using the two normal distributions defined in Section 3.3 and Section 3.4.2,

\[ P(X) = \text{Distribution of } \beta - \text{angle error measured by difference angle (Section 3.3)} \]

\[ P(X + Y) = \text{Distribution of total } \beta - \text{angle deviation from mean (Fig.7)} \]

\[ P(Y) \] is then,
If Equations [3] and [4] are utilized the mean and variance of this new distribution is:

\[ \mu_Y = \mu_{X+Y} - \mu_X = 0 - 0 = 0 \]
\[ \sigma_Y^2 = \sigma_{X+Y}^2 - \sigma_X^2 = 471 - 62 = 409 \]

The original $\beta$-angle measurements are now adjusted so that the distribution of their deviation about the mean $\beta$-angle matches that of $P(Y)$, instead of $P(X + Y)$. Each of the 190 measurements is adjusted individually so that the population fits the new normal distribution $P(Y)$. This is accomplished for each data point in the joint set as follows:

1) The difference between the measured $\beta$-angle and the $\beta$-angle that would have resulted in the mean vector is calculated (Section 3.4.2).

2) The normal distribution $P(X + Y)$ ($\mu = 0$ $\sigma^2 = 471$ for this example) is sampled using the result of step (1), which results in a probability between zero and one.

3) The probability from step (2) is passed through the inverse cumulative distribution function (quantile function) of the normal distribution $P(Y)$ ($\mu = 0$ $\sigma^2 = 409$ for this example), which results in a number closer to zero than step (1).

4) The $\beta$-angle is corrected closer to the mean value by the difference between the results from step (1) and step (3).

The corrected data set with the same mean $\beta$-angle is processed to create a new, tighter joint set. The original and new distribution of calculated joint dips from the processed data is presented in Figure 8.

![Figure 8. Dip distribution for the bedding joint set before and after the $\beta$-angle corrections](image)

### 3.5 Effect of reduced joint set scatter on probabilistic rock bench analysis

The probabilistic rock bench analysis program Backbreak (created by Call & Nicholas, Inc.) is utilized for the analysis. Back break is defined as the distance behind the crest that a vertical bench sloughs into the pit along
daylighted geologic structures. A distribution of predicted bench face angles is produced that is based upon structure orientations, lengths, spacings, shear strengths, and probability of occurrence.

The results of the probabilistic bench-scale analysis for the original bedding joint data set and the corrected set are presented in Figure 9. Both the bench face angle and the 95% reliability catch-bench width distributions are shown. The interramp slope angles that meet the design criteria discussed in Section 3.1 are presented in Table 2.

![Graphical results of probabilistic bench-scale analysis](image)

**Table 2.** Tabular results of probabilistic bench-scale analysis

<table>
<thead>
<tr>
<th></th>
<th>Original Data</th>
<th>Corrected Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>80% Reliability of Achieving &gt;7.6 meter catch-bench width</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data Set</td>
<td>Bench Face Angle</td>
<td>Interramp Angle</td>
</tr>
<tr>
<td>Original Data</td>
<td>60°</td>
<td>43°</td>
</tr>
<tr>
<td>Corrected Data</td>
<td>64°</td>
<td>45°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Original Data</th>
<th>Corrected Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>95% Reliability of Achieving &gt;0.0 meter catch-bench width</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data Set</td>
<td>Bench Face Angle</td>
<td>Interramp Angle</td>
</tr>
<tr>
<td>Original Data</td>
<td>39°</td>
<td>30°</td>
</tr>
<tr>
<td>Corrected Data</td>
<td>42°</td>
<td>32°</td>
</tr>
</tbody>
</table>

The change in achievable interramp angle is attributable to differing percentages of predicted joints that are steeper than the friction angle (22°). In the original data set 16% of the joints are at the friction angle or steeper, whereas in the new data set only 13% of the joints are at or steeper than the friction angle (Fig. 8). This 19% reduction in the number of joints steeper than the friction angle is enough to raise the design interramp angle by 2° for both bench design criteria.
It is important to note that for other cases the design interramp angle may increase or decrease as a result of the reduction in scatter, depending on the joint set’s location relative to the friction angle/kinematic feasibility boundary.

### 3.6 Limitations to difference angle error quantification

- Only error induced by the measuring of the *in situ* orientation, marking of the orientation, assembling of the core run, marking of the reference line, and the measuring of the DA is quantified.
- The DA can only be measured when two consecutive core orientations are obtained and are able to be compared across an assembled core run. This is generally difficult to accomplish in poor quality rock; therefore, DAs will be biased towards runs in good quality rock when the error is likely less.
- In order to statistically remove error from the data, it must be assumed that the calculated mean vector of a joint set is correctly identified through oriented core drilling and that it does not vary significantly with depth.
- The distribution of DAs and the distribution of total variation from the mean β-angle $P(X + Y)$ must fit the normal distribution.
- A joint measurement cannot be corrected to its location before the introduction of error. The correction must be treated statistically. Therefore, error correction is only useful for probabilistic analysis.

### 3.7 When are these methods useful?

It is always useful to quantify the amount of the error measured by the DA induced in the β-angle, as described in *Section 3.3*. However, statistically removing the error from a joint set data population distribution is only useful when performing probabilistic bench-scale analysis and one of the following conditions are met:

- When the joint set is in the plane shear orientation and the friction circle intersects the set. If a higher or lower percentage of the measured joint set population will be above or below the friction angle as a result of the reduction in data scatter, achievable interramp angles will likely change.
- When the joint set is intersected by the boundary of kinematic feasibility (typically ±20° for plane shear sliding). If a higher or lower percentage of the measured joint set will be kinematically feasible for sliding as a result of the reduction in data scatter, achievable interramp angles will likely change.

Statistical β-angle error removal (*Section 3.4.3*) should only be performed when the α-angle that yields the mean vector of the joint set is less than 80°, as changes in the β-angle are very insignificant for large α-angles (Fig. 3a, Fig. 6b).

### 4 Conclusions

It is essential to evaluate the quality and accuracy of the various sources of geologic data that engineers use to design rock slopes. Oriented core drilling is commonly used to collect joint orientation data for rock slope design, yet very little has been proposed regarding how to measure or quantify errors induced during the orientation process. The difference angle measurement allows quantification of the distribution of some of the error induced during core orientation. The difference angle is used to calculate the measured mean β-angle error and the measured mean error in the final converted data for runs in which difference angles were taken. The dispersion about the mean β-angle is modified to remove the added variation induced by the measured error, and can result in steeper or shallower interramp angles when performing probabilistic rock bench analysis. Future work on this topic may include studies investigating the effect of joint waviness on oriented-core data, the accuracy and repeatability of various down-hole survey tools, or direct comparison between oriented-core and surface mapping data.
5 References


6 Appendix

6.1 Joint set spacing estimation from oriented core

It is often difficult to obtain accurate estimations of joint set spacing form oriented-core data (Park & West 2002). Joint set spacing estimations from oriented drilling are typically wider than those obtained from surface mapping. Two oriented core holes were drilled in the siltstone described in Section 3.1 directly behind recently exposed mining faces. Cell mapping to collect bedding joint set data was conducted 250 meters from the location of oriented core drilling. Good spacing estimation correlation between the two data sets was achieved by processing the data as follows:

1) Calculate the true spacing between joints of the same set, but only if there is continuous orientation in the interval in between the joints (i.e. no lost orientation zones); otherwise do not consider the interval.

2) Calculate the median of the spacing measurements obtained from step (1)

6.2 Solution to correct apparent joint spacing to true spacing from drill core

After Priest (1993),

\[ \psi = R \left| \cos(\theta_1 - \theta_2 + 180^\circ) \cos(90^\circ - \delta_1) \cos \delta_2 - \sin(90^\circ - \delta_1) \sin \delta_2 \right| \]  

\[ \psi = \text{True joint spacing} \quad R = \text{Drill interval between joints (apparent spacing)} \]

\[ \theta_1 = \text{Dip direction of joint plane} \quad \delta_1 = \text{Dip of joint plane} \]

\[ \theta_2 = \text{Azimuth of drill hole} \quad \delta_2 = \text{Plunge of drill hole (negative for downward drilling)} \]